HiX2010, Jefferson Lab
October 15, 2010

Quark-hadron duality in structure functions

Wally Melnitchouk

Jefferson Lab

Outline

- Introduction / historical context
- Duality in QCD
  - → resonances & higher twists
- Local duality
  - → truncated moments
  - → insights from models
- Implications for semi-inclusive DIS
- Summary

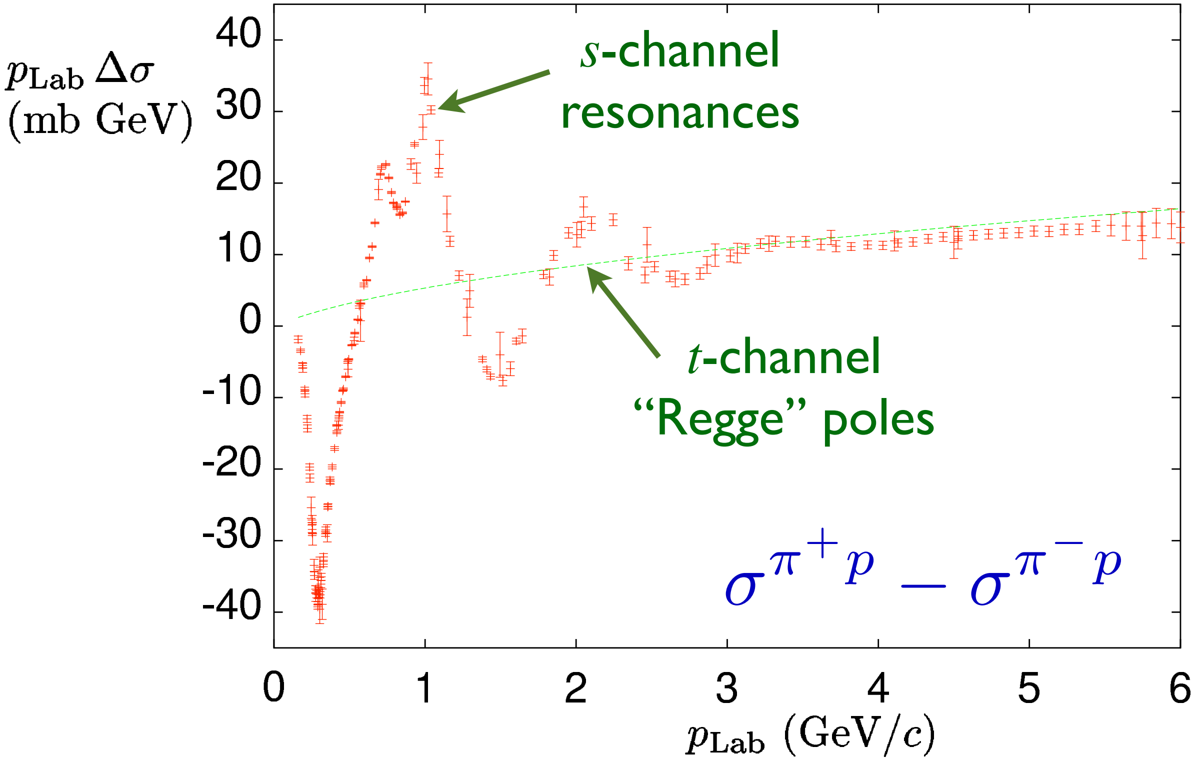
Quark-hadron duality

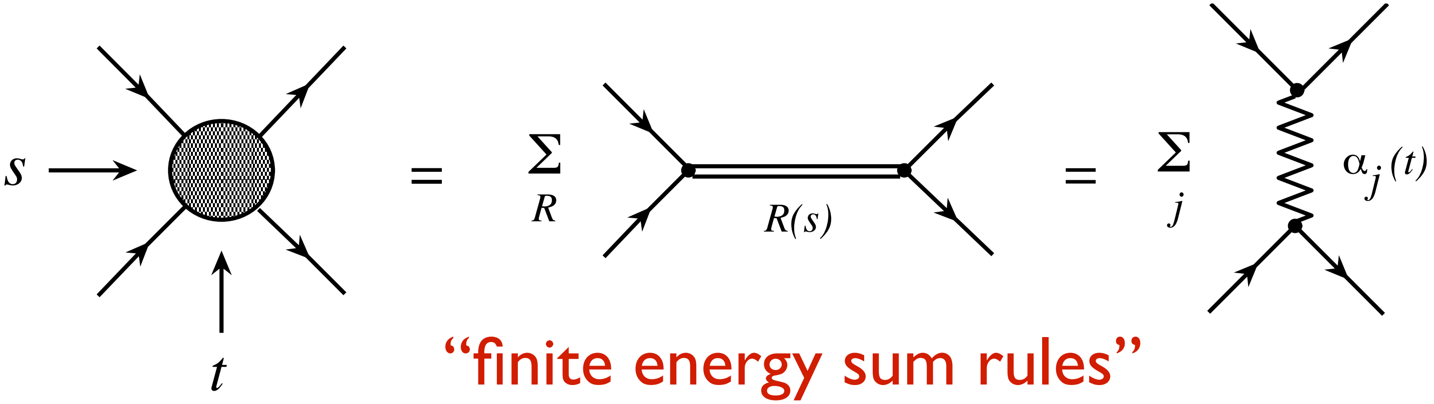
Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

Duality in hadron-hadron scattering



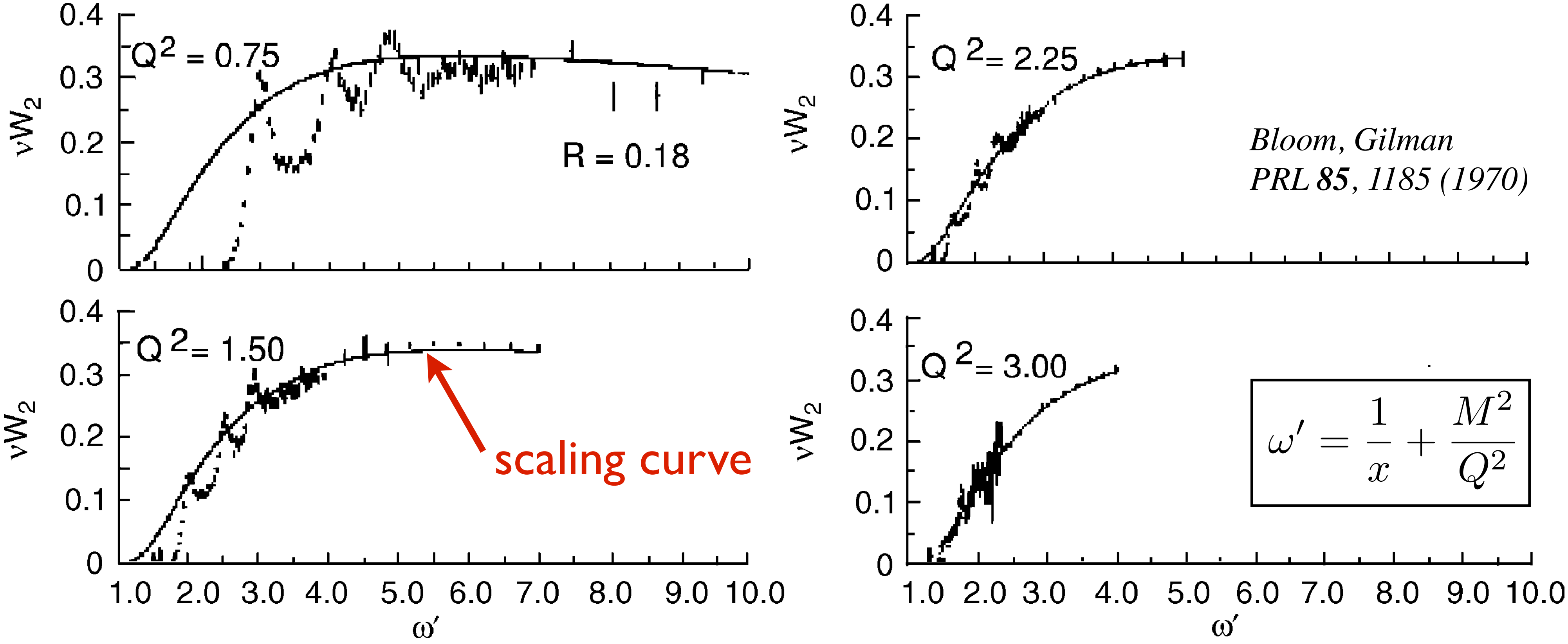


"finite energy sum rules"

Igi (1962), Dolen, Horn, Schmidt (1968)

Duality in electron-hadron scattering

“Bloom-Gilman duality”



scaling curve

Bloom, Gilman
PRL ***85****, 1185 (1970)*

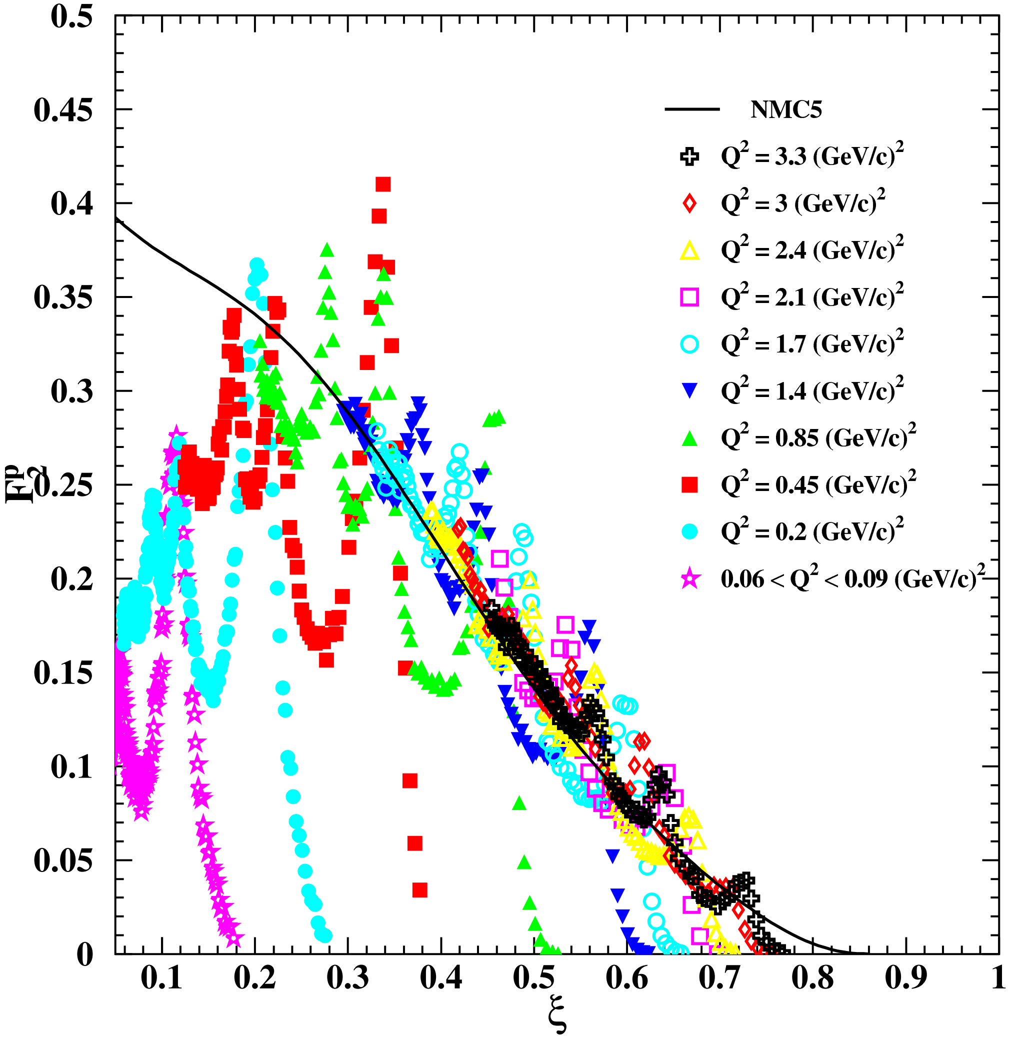
$$\boxed{\omega' = \frac{1}{x} + \frac{M^2}{Q^2}}$$

→ finite energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

“hadrons” “quarks”

Duality in electron-hadron scattering



Niculescu et al., PRL ***85****, 1182 (2000)*

average over

(strongly Q^2 dependent)

resonances

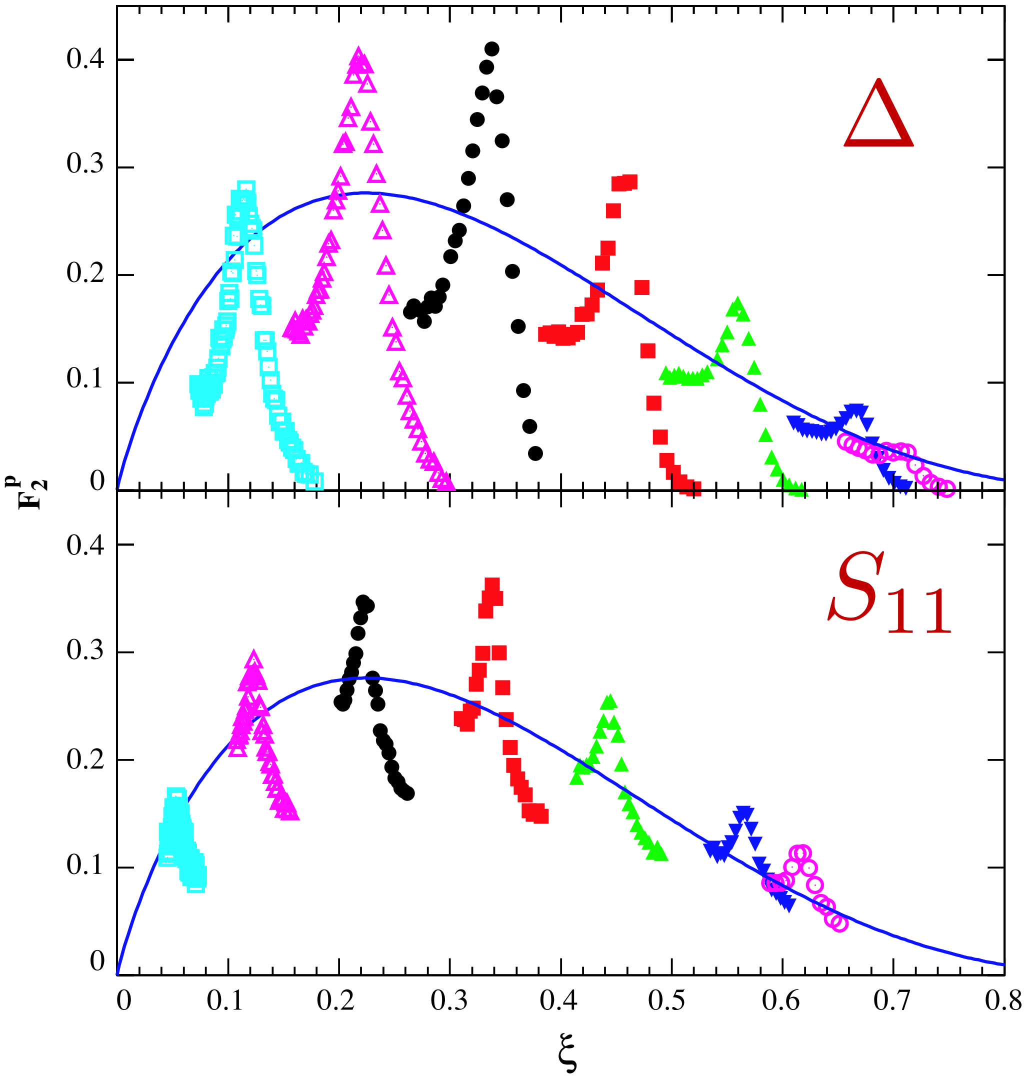
$\approx Q^2$ independent

scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Duality in electron-hadron scattering



→ also exists "*locally*" in individual resonance regions

Duality in QCD

(“global duality”)

Duality in QCD

- Operator product expansion

→ expand *moments* of structure functions in powers of $1/Q^2$

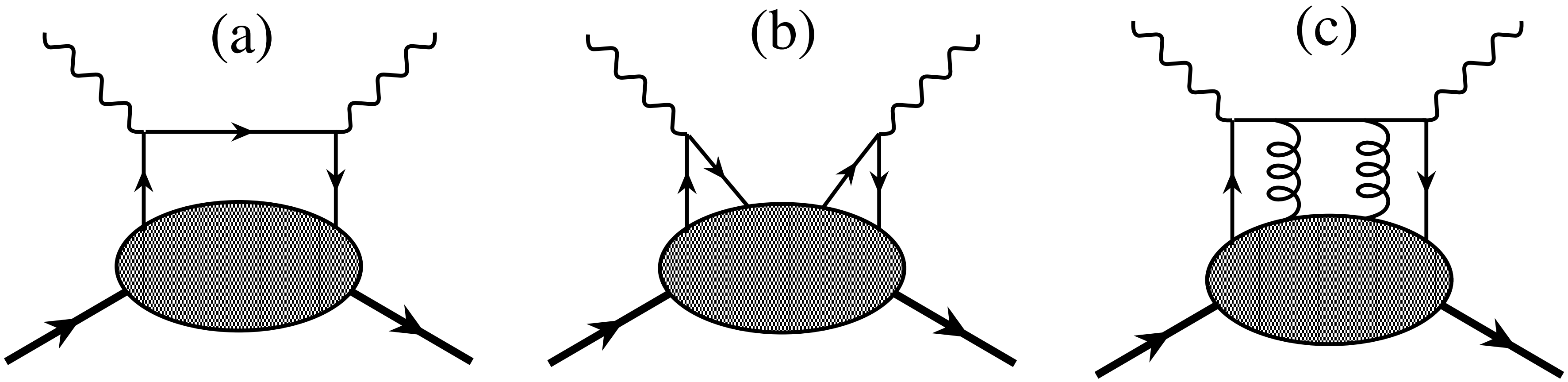
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Duality in QCD



$\tau = 2$

single quark scattering

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi$

$\tau > 2$

qq and qg correlations

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\quad \bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD

- Operator product expansion

  $\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \; x^{n-2} \; F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

- Operator product expansion

  → expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

- Duality ⟷ suppression of higher twists

de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

Resonances & higher twists

- Much of recent new data is in *resonance* region, $W < 2$ GeV

  → *common wisdom:* pQCD analysis not valid in resonance region

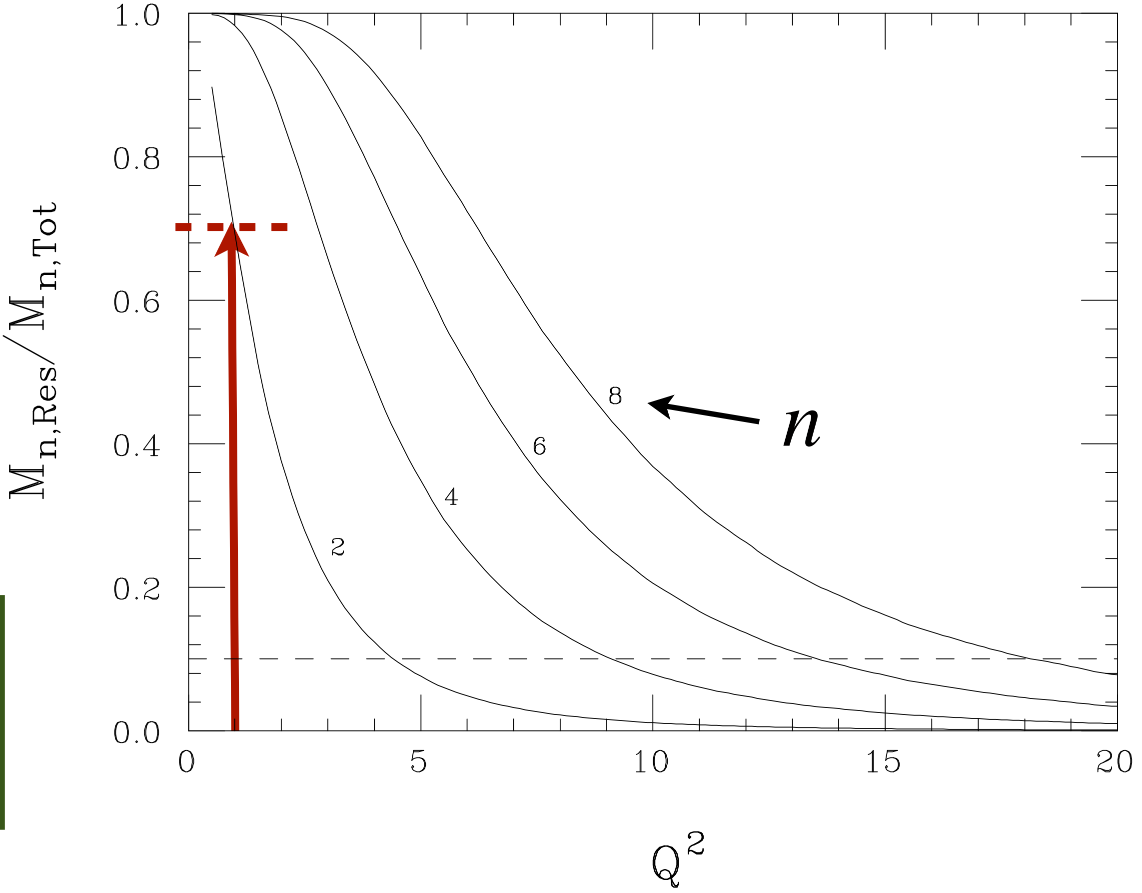
  (→ talks of Owens, Accardi / CTEQX)

  → *in fact:* partonic interpretation of moments *does* include resonance region

- Resonances are an *integral part* of DIS structure functions!

  → implicit role of quark-hadron duality

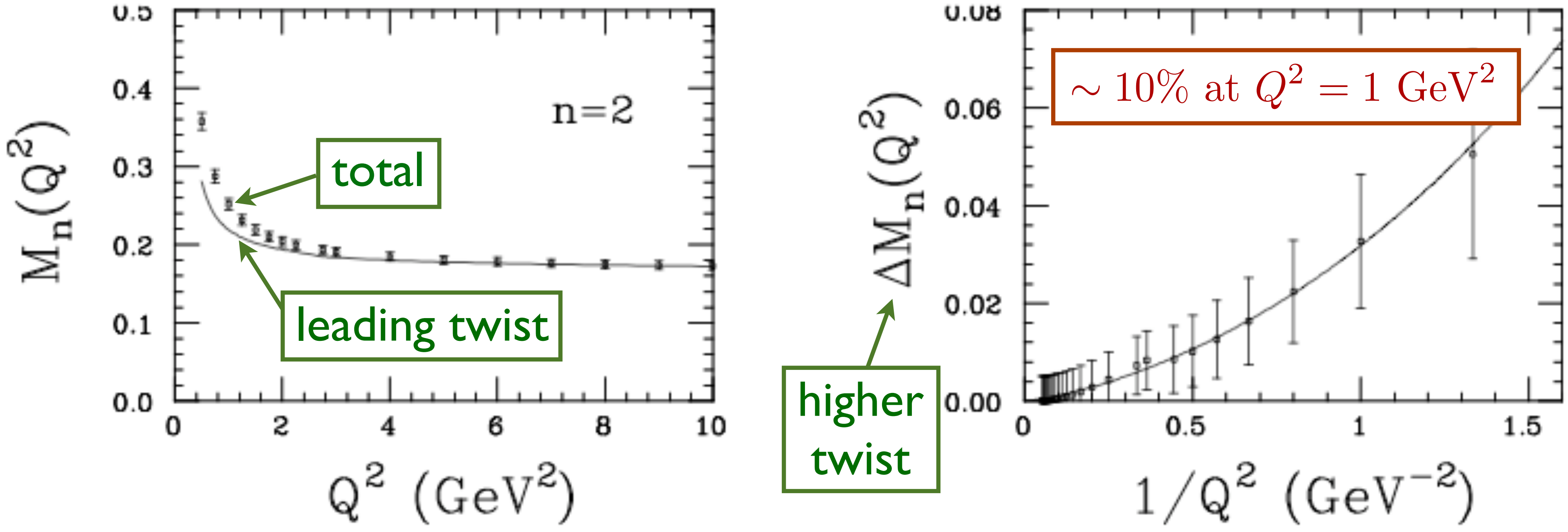
Resonances & higher twists



relative contribution of resonance region to n-th moment

➡ At $Q^2 = 1\ \mathrm{GeV}^2$, $\sim$ 70% of lowest moment of F_2^p comes from $W < 2\ \mathrm{GeV}$

Resonances & higher twists



total

leading twist

higher twist

$\sim 10\%$ at $Q^2 = 1$ GeV^2

➡ BUT resonances and DIS continuum conspire to produce only $\sim$ 10% *higher twist* contribution!

Ji, Unrau, PRD ***52****, 72 (1995)*

Resonances & higher twists

- total higher twist *small* at $Q^2 \sim 1 - 2\ \mathrm{GeV}^2$

- on average, nonperturbative interactions between quarks and gluons not dominant at these scales

- suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

- OPE does not tell us *why* higher twists are small

→ need more detailed information
(*e.g.* about individual resonances & their cancellations)
to understand behavior dynamically

Local Duality:

truncated moments

Truncated moments

- complete moments can be studied via twist expansion

  $\longrightarrow$ Bloom−Gilman duality has a precise meaning

  (*i.e.*, duality violation = higher twists)

- rigorous connection between local duality & QCD difficult

  $\longrightarrow$ need prescription for how to average over resonances

- *truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

where modified splitting function is

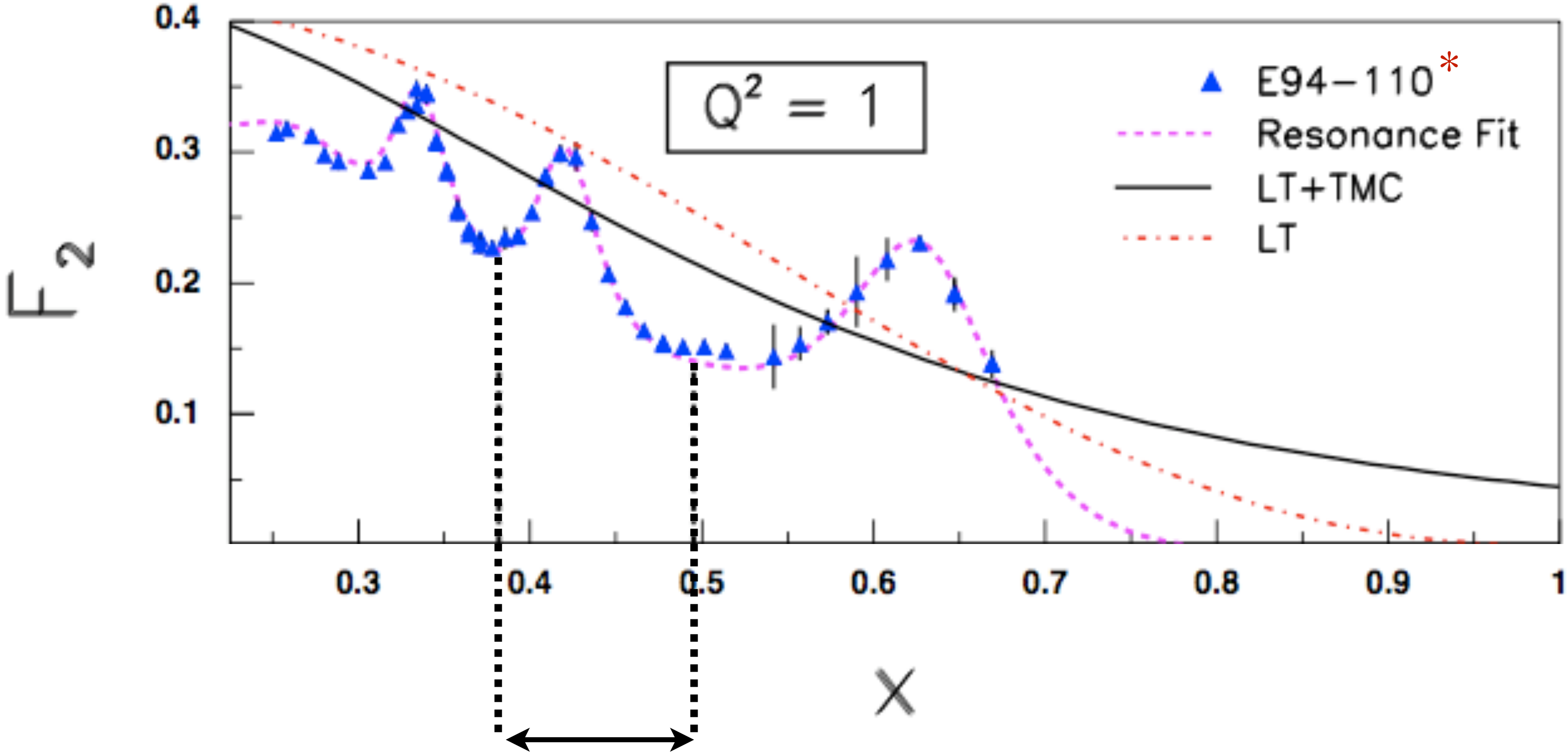
Forte, Magnea, PLB ***448****, 295 (1999)*
Kotlorz, Kotlorz, PLB ***644****, 284 (2007)*

$$P'_{(n)}(z, \alpha_s) = z^n \; P_{NS,S}(z, \alpha_s)$$

→ can follow evolution of *specific resonance (region)* with Q^2 in pQCD framework!

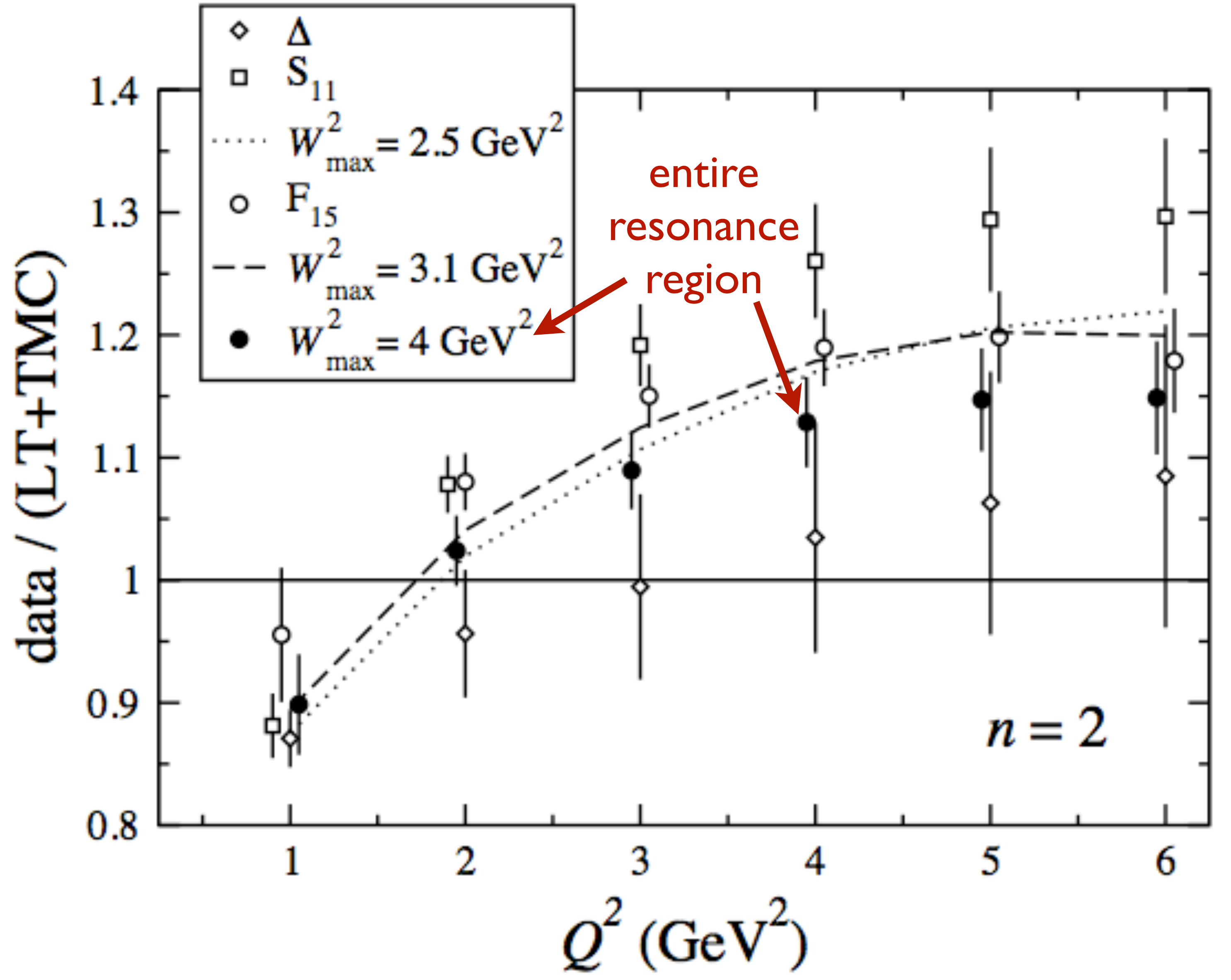
→ suitable when complete moments not available

F_2^p resonance spectrum



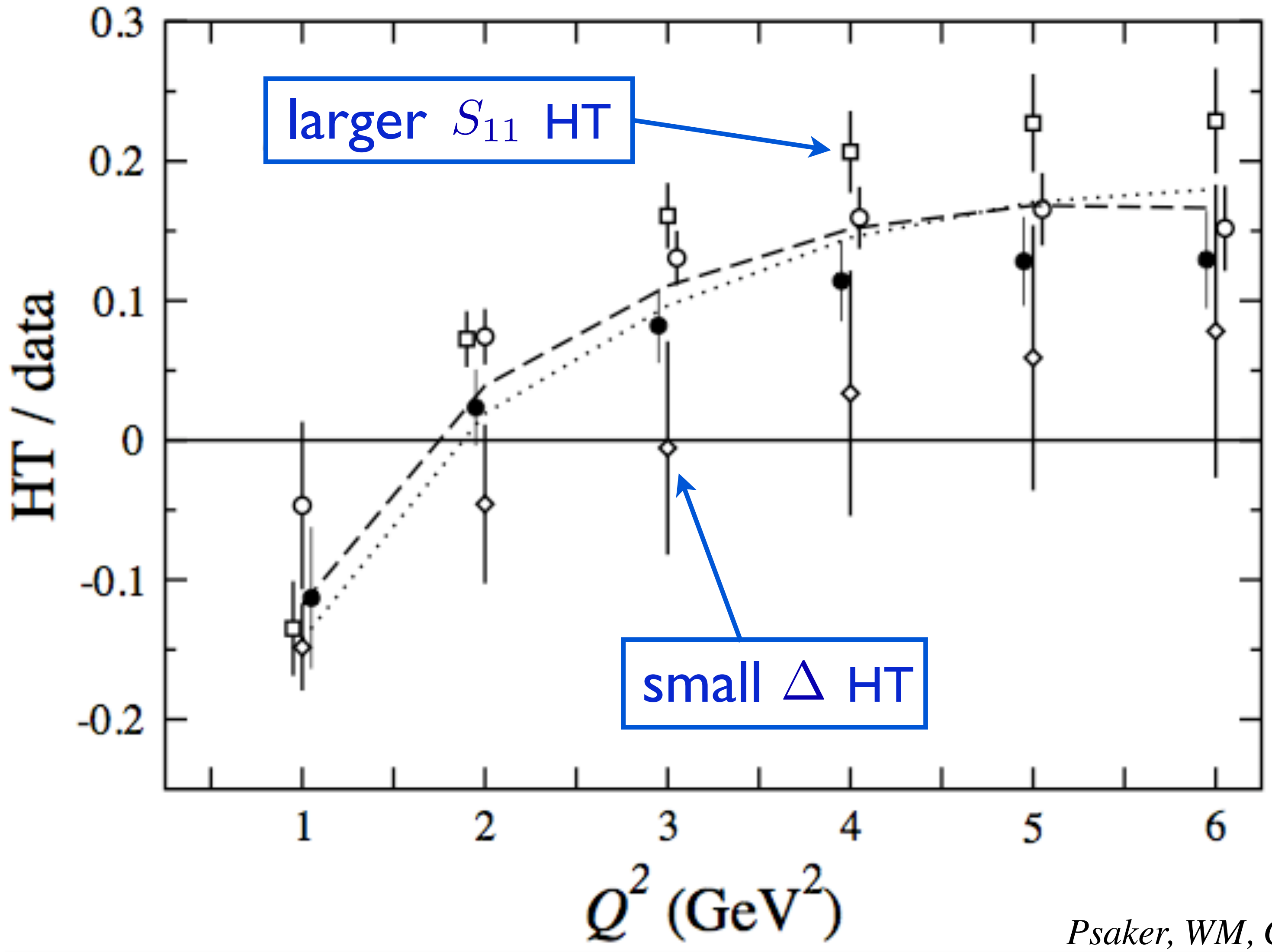
* JLab Hall C

how much of this region is *leading twist* ?



entire resonance region

Psaker, WM, Christy, Keppel
PRC ***78****, 025206 (2008)*



larger S_{11} HT

small Δ HT

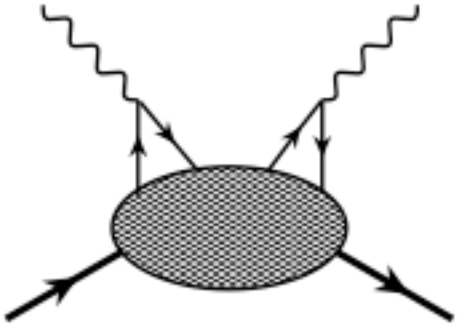
Psaker, WM, Christy, Keppel
*PRC **78**, 025206 (2008)*

→ higher twists $< 10{-}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Local Duality:
insights from models

Is duality in the proton a coincidence?

- consider model with symmetric nucleon wave function



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i\neq j} e_i\, e_j \sim \Big(\underbrace{\sum_i e_i}_{\uparrow\ \textit{coherent}}\Big)^2 - \underbrace{\sum_i e_i^2}_{\uparrow\ \textit{incoherent}}$$

- *proton* $\mathrm{HT} \sim 1 - \left(2\times\frac{4}{9}+\frac{1}{9}\right) = 0\ !$

- *neutron* $\mathrm{HT} \sim 0 - \left(\frac{4}{9}+2\times\frac{1}{9}\right) \neq 0$

Brodsky (HiX'00)
hep-ph/0006310

⇒ need to test duality in proton *and* neutron!

- How can the *square of a sum* become the *sum of squares*?

→ in *hadronic* language, duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

Close, Isgur, PLB ***509****, 81 (2001)*

→ in NR Quark Model, even and odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

- assume magnetic coupling of photon to quarks
  (better approximation at high Q^2)
- in this limit Callan-Gross relation valid $F_2 = 2xF_1$
- structure function given by squared sum of transition FFs

$$F_1(\nu,\vec{q}^2) \sim \sum_R |F_{N\to R}(\vec{q}^2)|^2 \delta(E_R - E_N - \nu)$$

■ How can the *square of a sum* become the *sum of squares*?

→ in *hadronic* language, duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

Close, Isgur, PLB ***509****, 81 (2001)*

→ in NR Quark Model, even and odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

Close, WM, PRC ***68****, 035210 (2003)*

- SU(6) limit ➡ $\lambda = \rho$

  → relative strengths of $N \to N^*$ transitions:

$SU(6):$	$[\mathbf{56},0^+]^2\mathbf{8}$	$[\mathbf{56},0^+]^4\mathbf{10}$	$[\mathbf{70},1^-]^2\mathbf{8}$	$[\mathbf{70},1^-]^4\mathbf{8}$	$[\mathbf{70},1^-]^2\mathbf{10}$	$total$
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

  → $\dfrac{F_1^n}{F_1^p} = \dfrac{2}{3}$ as in quark-parton model (for $u=2d$) !

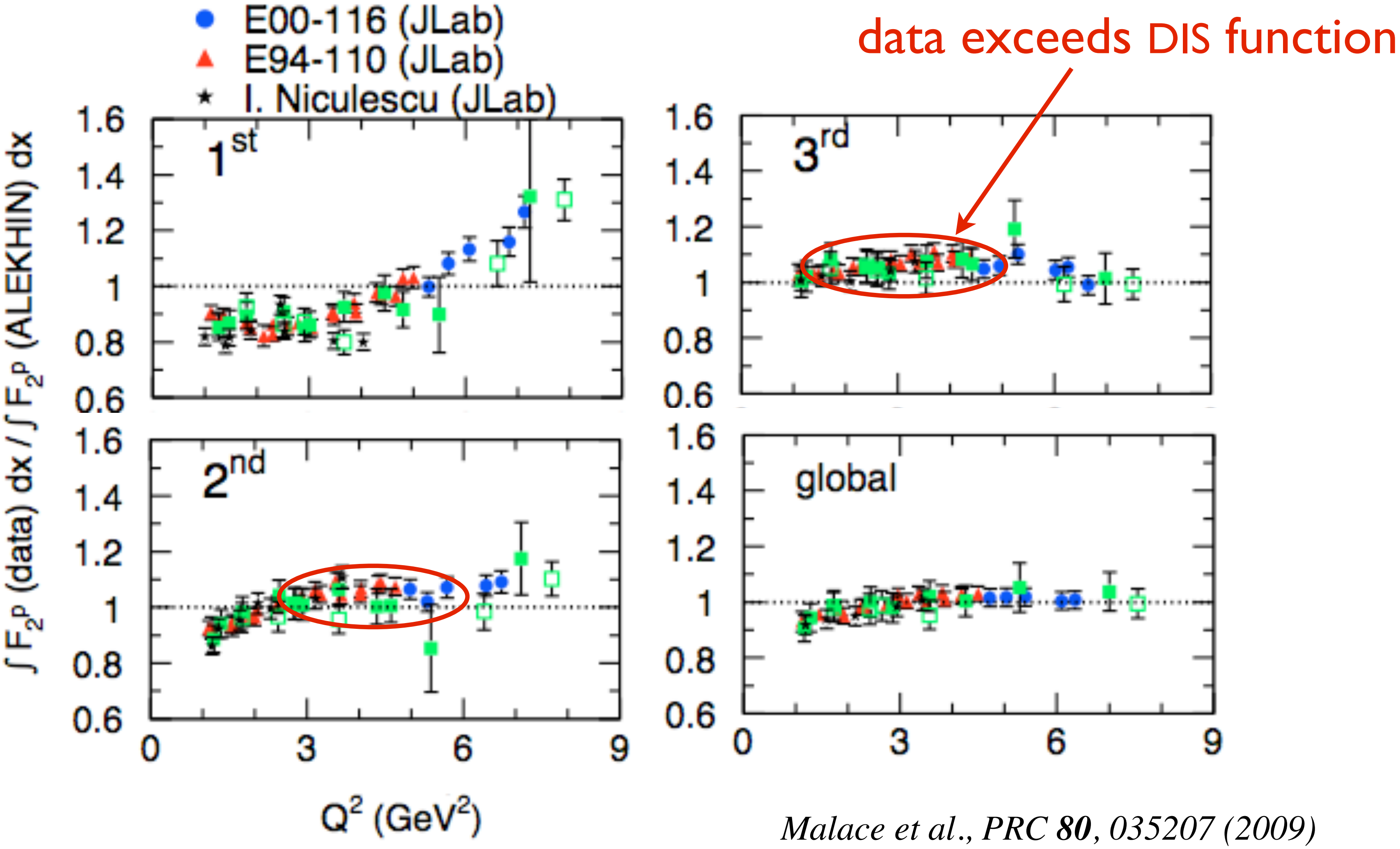
- proton sum saturated by lower-lying resonances

  → expect duality to appear *earlier* for p than n

Close, WM, PRC ***68****, 035210 (2003)*

Comparison with data

- proton data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)



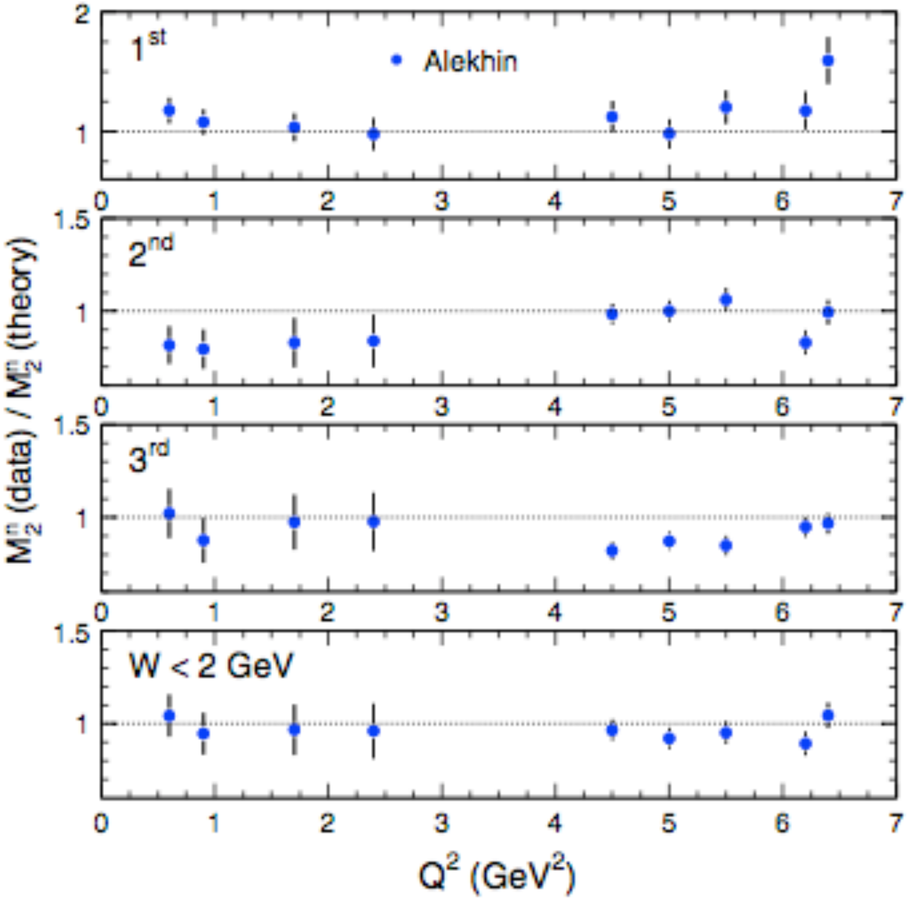
data exceeds DIS function

Malace et al., PRC ***80****, 035207 (2009)*

(⟶ talk of Malace)

Comparison with data

- neutron data predicted to lie *below* DIS function in 2nd region



→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

→ *locally*, violations of duality in resonance regions < 15−20% (largest in Δ region)

→ *globally*, violations < 10%

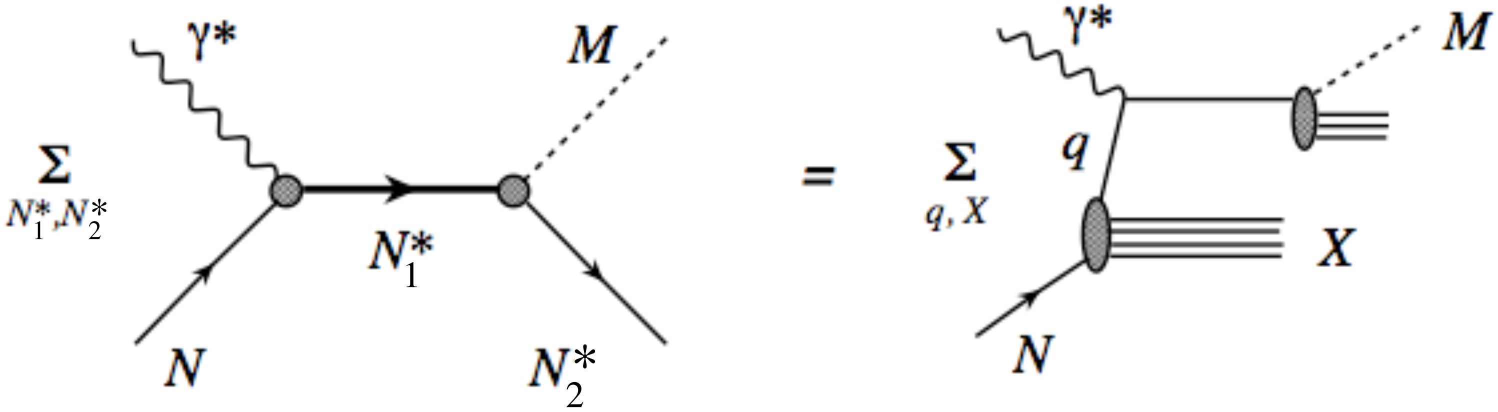
Malace, Kahn, WM, Keppel
*PRL **104**, 102001 (2010)*

⇒ duality is *not accidental*, but a general feature of resonance−scaling transition!

Duality in Semi-Inclusive Meson Production

- Duality expected to work better for inclusive observables (*e.g.* structure functions)

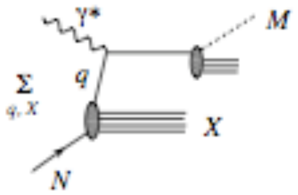
- Hypothesis: equivalent descriptions of semi-inclusive meson production afforded by scattering *via* partons *or* N^* excitations



Afanasev, Carlson, Wahlquist, PRD ***62****, 074011 (2000)*
Hoyer, arXiv:hep-ph/0208190

→ test hypothesis with *models* and *data*

Partonic description

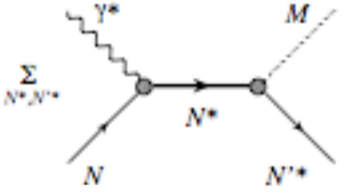


$$\mathcal{N}_N^\pi(x,z) \;=\; e_u^2\, u^N(x)\, D_u^\pi(z) \;+\; e_d^2\, d^N(x)\, D_d^\pi(z)$$

$q \longrightarrow \pi$ fragmentation function

$z = E_\pi/\nu$ fractional energy carried by pion

Hadronic description

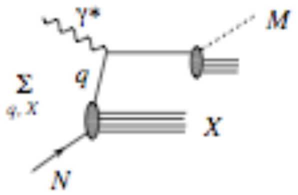


$$\mathcal{N}_N^\pi(x,z) \;=\; \sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N\to N_1^*}(Q^2, M_1^*)\; \mathcal{D}_{N_1^*\to N_2^*\pi}(M_1^*, M_2^*) \right|^2$$

transition
form factor

decay function

Partonic description

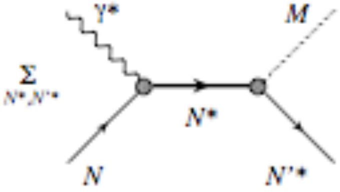


$$\mathcal{N}_N^{\pi}(x,z) \;=\; e_u^2\, u^N(x)\, D_u^{\pi}(z) \;+\; e_d^2\, d^N(x)\, D_d^{\pi}(z)$$

→ ratios given by quark charges

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} \;=\; \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} \;=\; \frac{e_u^2}{e_d^2} \;=\; 4$$

Hadronic description



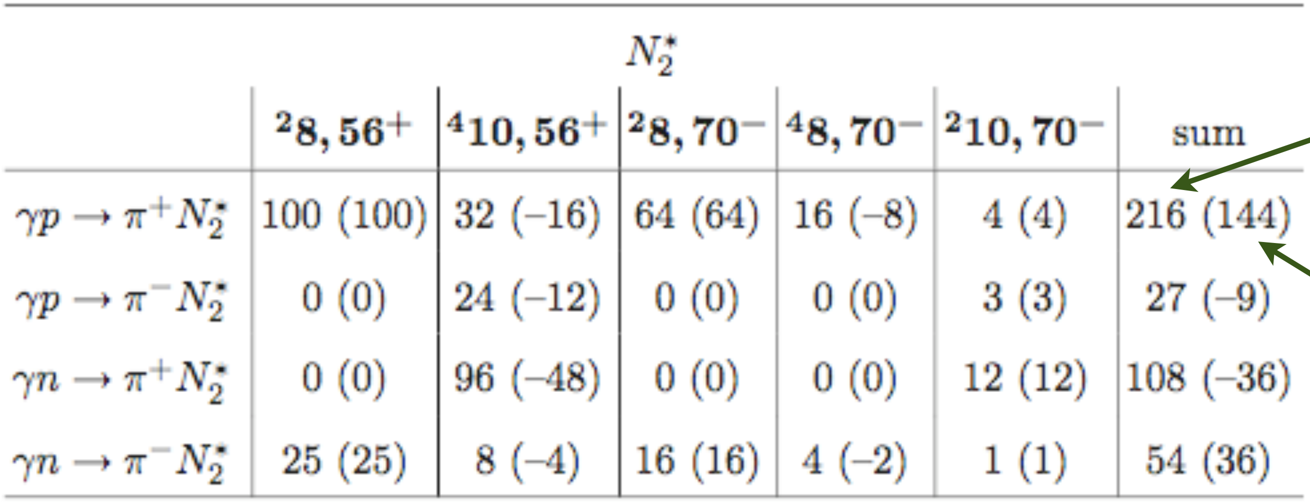
→ magnetic interaction operator for $\gamma N \to N_1^*$

$$\sum_i e_i\, \sigma_i^{+}$$

→ pion emission operator for $N_1^* \to N_2^*\, \pi^{\pm}$

$$\sum_i \tau_i^{\mp}\, \sigma_{zi}$$

- Relative probabilities $\mathcal{N}_N^\pi$ in SU(6) symmetric quark model (summed over N_1^*)



	N_2^*					
	$^2\mathbf{8}, \mathbf{56}^+$	$^4\mathbf{10}, \mathbf{56}^+$	$^2\mathbf{8}, \mathbf{70}^-$	$^4\mathbf{8}, \mathbf{70}^-$	$^2\mathbf{10}, \mathbf{70}^-$	sum
$\gamma p \to \pi^+ N_2^*$	100 (100)	32 (–16)	64 (64)	16 (–8)	4 (4)	216 (144)
$\gamma p \to \pi^- N_2^*$	0 (0)	24 (–12)	0 (0)	0 (0)	3 (3)	27 (–9)
$\gamma n \to \pi^+ N_2^*$	0 (0)	96 (–48)	0 (0)	0 (0)	12 (12)	108 (–36)
$\gamma n \to \pi^- N_2^*$	25 (25)	8 (–4)	16 (16)	4 (–2)	1 (1)	54 (36)

spin-averaged

spin-dependent

Close, WM, PRC ***79****, 055202 (2009)*

- π^-/π^+ ratios for p and n targets (summing over N_2^*)

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{8}, \qquad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{1}{2}$$

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{2}, \qquad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = 4$$

- Consistent with parton model in SU(6) limit, $d/u=1/2$

- For *spin-dependent* ratios (e & N longitudinally polarized)

$$\frac{\Delta\mathcal{N}_p^{\pi^-}}{\Delta\mathcal{N}_p^{\pi^+}} = -\frac{1}{16}\,, \qquad \frac{\Delta\mathcal{N}_n^{\pi^-}}{\Delta\mathcal{N}_n^{\pi^+}} = -1$$

$$\frac{\Delta\mathcal{N}_p^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{2}{3}\,, \qquad \frac{\Delta\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^-}} = -\frac{1}{3}$$

$$\frac{\Delta\mathcal{N}_n^{\pi^+}}{\mathcal{N}_n^{\pi^+}} = -\frac{1}{3}\,, \qquad \frac{\Delta\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{2}{3}$$

- Consistent with parton model ratios

$$\Delta u/u = 2/3\,, \;\; \Delta d/d = -1/3\,, \;\; \Delta d/\Delta u = -1/4$$

- Inclusive results recovered by summing over π^+ & π^-

$$\frac{\mathcal{N}_n^{\pi^+ + \pi^-}}{\mathcal{N}_p^{\pi^+ + \pi^-}} = \frac{F_1^n}{F_1^p} = \boxed{\frac{2}{3}}$$

$$\frac{\Delta\mathcal{N}_p^{\pi^+ + \pi^-}}{\mathcal{N}_p^{\pi^+ + \pi^-}} = \frac{g_1^p}{F_1^p} = \boxed{\frac{5}{9}}, \qquad \frac{\Delta\mathcal{N}_n^{\pi^+ + \pi^-}}{\mathcal{N}_n^{\pi^+ + \pi^-}} = \frac{g_1^n}{F_1^n} = \boxed{0}$$

- SU(6) symmetry may be valid at $x \sim 1/3$, but is (badly) broken at large x

- Color-magnetic interaction

  → suppression of transitions to states with S=3/2

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{56}\,, \qquad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{7}{2}$$

  → consistent with d/u=1/14 at parton level

- Scalar diquark dominance

  → suppression of symmetric (λ) component of wfn.

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = 0\,, \qquad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = 0\,, \qquad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{4}$$

  → consistent with d/u=0 at parton level

- SU(6) symmetry may be valid at $x \sim 1/3$, but is (badly) broken at large x

- Helicity conservation

→ suppression of helicity-$3/2$ amplitude

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{20}\,, \qquad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{5}{4}\,, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{5}$$

→ consistent with $d/u=1/5$ at parton level

- SU(6) symmetry may be valid at $x \sim 1/3$, but is (badly) broken at large x

- Helicity conservation

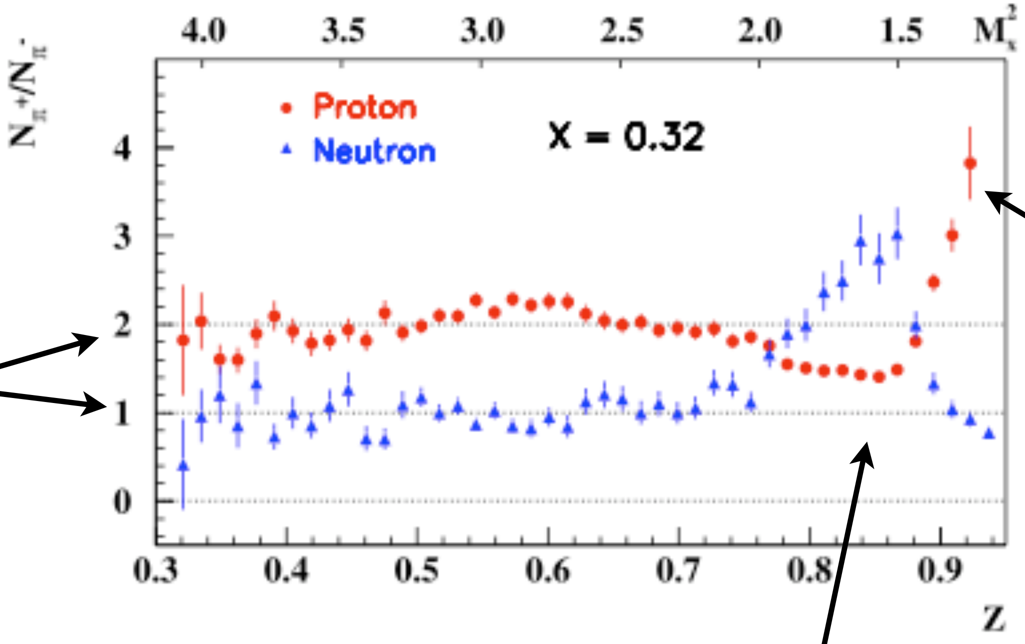
→ suppression of helicity-$3/2$ amplitude

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{20}, \qquad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{5}{4}, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{5}$$

→ consistent with $d/u{=}1/5$ at parton level

⇒ All three scenarios consistent with duality!

■ Comparison with data (JLab Hall C)

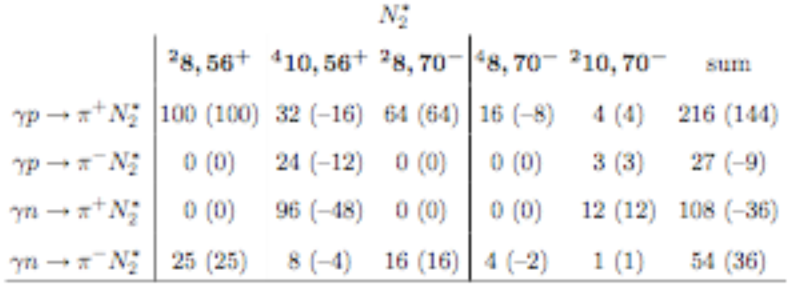


expect
p ratio
$\gg n$ ratio

N elastic
p ratio $\gg n$ ratio

Δ region
n ratio $\gg p$ ratio

Navasardyan et al., *PRL* ***98***, *022001 (2007)*

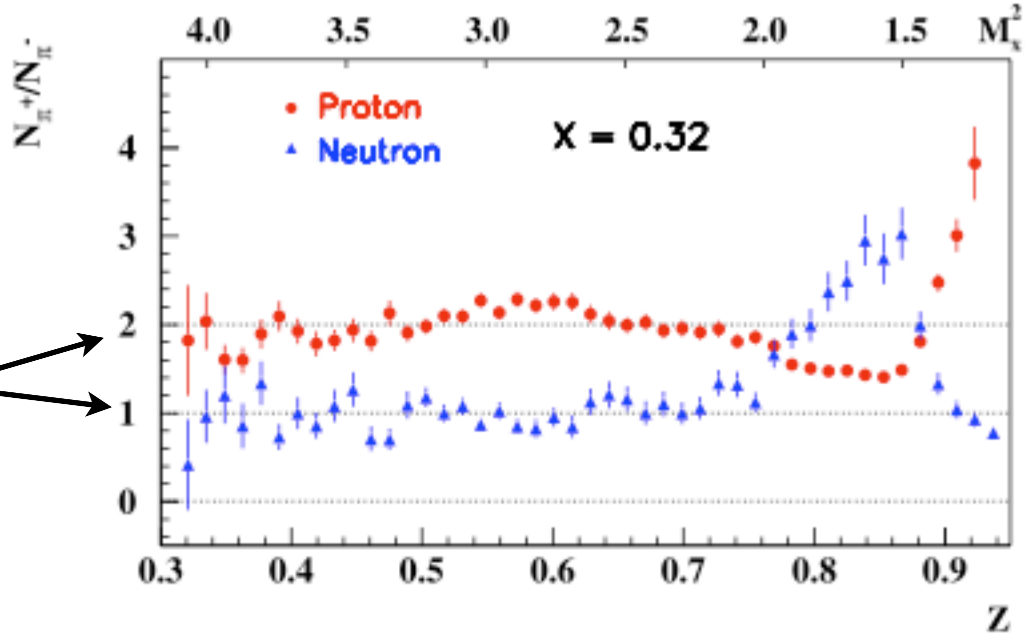


	N_2^*					
	$^2\mathbf{8},\mathbf{56}^+$	$^4\mathbf{10},\mathbf{56}^+$	$^2\mathbf{8},\mathbf{70}^-$	$^4\mathbf{8},\mathbf{70}^-$	$^2\mathbf{10},\mathbf{70}^-$	sum
$\gamma p \to \pi^+ N_2^*$	100 (100)	32 (−16)	64 (64)	16 (−8)	4 (4)	216 (144)
$\gamma p \to \pi^- N_2^*$	0 (0)	24 (−12)	0 (0)	0 (0)	3 (3)	27 (−9)
$\gamma n \to \pi^+ N_2^*$	0 (0)	96 (−48)	0 (0)	0 (0)	12 (12)	108 (−36)
$\gamma n \to \pi^- N_2^*$	25 (25)	8 (−4)	16 (16)	4 (−2)	1 (1)	54 (36)

Comparison with data (JLab Hall C)

smaller than SU(6) predictions

(secondary fragmentation)



Navasardyan et al., *PRL* ***98***, *022001 (2007)*

More quantitative comparison requires secondary fragmentation

$$\frac{\mathcal{N}_d^{\pi^+}}{\mathcal{N}_d^{\pi^-}} = \frac{4+R}{4R+1}$$

$$R \equiv \overline{D}/D$$

$$D_d^{\pi^+} = D_u^{\pi^-}$$

"unfavored"

$$D_u^{\pi^+} = D_d^{\pi^-}$$

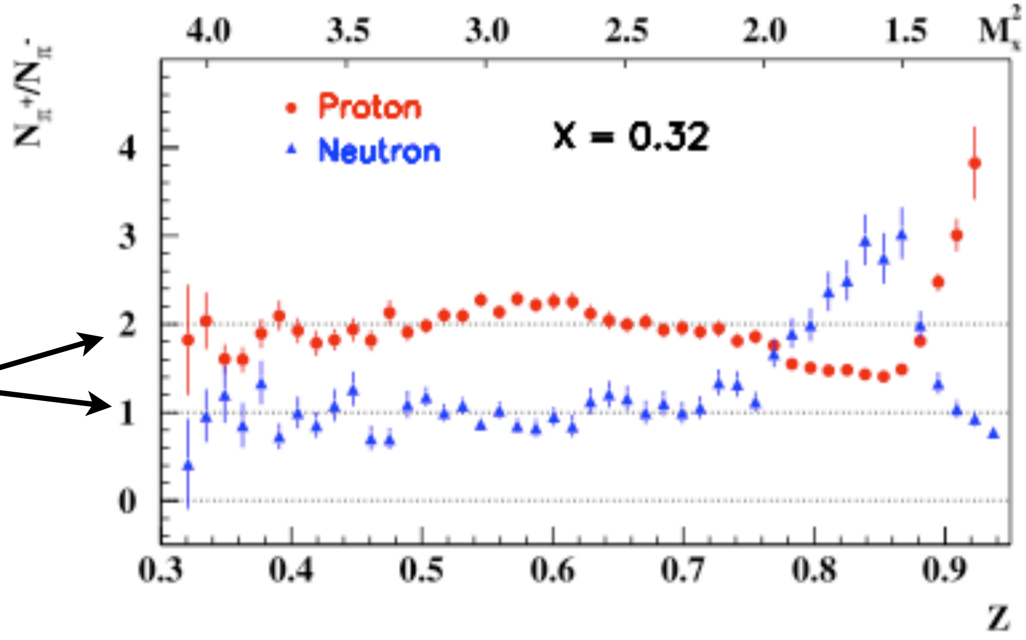
"favored"

$z \to 1$

- Comparison with data (JLab Hall C)

smaller than SU(6) predictions

(secondary fragmentation)



Navasardyan et al.,
PRL ***98***, *022001 (2007)*

- More quantitative comparison requires secondary fragmentation

- Target & (produced) hadron mass corrections recently computed for first time

Accardi, Hobbs, WM
JHEP ***0911***,*084 (2009)*

(⟶ talk of Accardi)

Summary

- Remarkable confirmation of quark-hadron duality in *proton* *and* *neutron* structure functions
  - → duality violating higher twists ~ 10–15% in few-GeV range
  - → duality is *not* due to accidental cancellations of quark charges

- Progress in deconstructing *local* duality
  - → evolution of truncated moments in QCD
  - → insight from quark models into how resonance cancellations may arise in nature

- First glimpses of duality in semi-inclusive pion production
  - → understanding degree to which duality "works" would greatly aid extraction of nucleon's partonic structure

The End